VOLUME 78 SEPARATE No. 124

Rochester Public Library
APR 2
115 South Avenue
ROCHESTER 4, N. Y.

PROCEEDINGS

AMERICAN SOCIETY OF CIVIL ENGINEERS

APRIL, 1952

INFLUENCE LINES BY CORRECTIONS TO AN ASSUMED SHAPE

By James P. Michalos, A. M. ASCE, and Edward N. Wilson, J. M. ASCE

STRUCTURAL DIVISION

Copyright 1952 by the AMERICAN SOCIETY OF CIVIL ENGINEERS
Printed in the United States of America

Headquarters of the Society
33 W. 39th St.
New York 18, N.Y.

PRICE $0.50 PER COPY

R 620.6
A 512p

GUIDEPOST FOR TECHNICAL READERS

"Proceedings-Separates" of value or significance to readers in various fields are here listed, for convenience, in terms of the Society's Technical Divisions. Where there seems to be an overlapping of interest between Divisions, the same Separate number may appear under more than one item.

Technical Division	*Proceedings-Separate Number*
Air Transport	42, 43, 48, 52, 60, 93, 94, 95, 100, 103, 104, 108, 121 (Discussion: D-XXVIII, D-7, D-16, D-18, D-23, D-43)
City Planning	58, 60, 62, 64, 93, 94, 99, 101, 104, 105, 115 (Discussion: D-16, D-23, D-43, D-60, D-62, D-65)
Construction	43, 50, 55, 71, 92, 94, 103, 108, 109, 113, 117, 121, 126 (Discussion: D-3, D-8, D-17, D-23, D-36, D-40, D-71)
Engineering Mechanics	41, 49, 51, 54, 56, 59, 61, 88, 89, 96, 116, 122, 124, 125, 126 (Discussion: D-5, D-XXIII, D-XXV, D-18, D-24, D-33, D-34 D-49, D-54, D-61)
Highway	43, 44, 48, 58, 70, 100, 105, 108, 113, 120, 121 (Discussion: D-XXVIII, D-7, D-13, D-16, D-23, D-60)
Hydraulics	50, 55, 56, 57, 70, 71, 78, 79, 80, 83, 86, 92, 96, 106, 107, 110, 111, 112, 113, 116, 120, 123 (Discussion: D-XXVII, D-9, D-11, D-19, D-28, D-29, D-56, D-70, D-71)
Irrigation and Drainage	46, 47, 48, 55, 56, 57, 67, 70, 71, 87, 88, 90, 91, 96, 97, 98, 99, 102, 106, 109, 110, 111, 112, 114, 117, 118, 120 (Discussion: D-XXIII, D-3, D-7, D-11, D-17, D-19, D-25-K, D-29, D-30, D-38, D-40, D-44, D-47, D-57, D 70, D-71)
Power	48, 55, 56, 69, 71, 88, 96, 103, 106, 109, 110, 117, 118, 120 (Discussion: D-XXIII, D-2, D-3, D-7, D-11, D-17, D-19, D-25-K, D-30, D-38, D-40, D-44, D-70, D-71)
Sanitary Engineering	55, 56, 87, 91, 96, 106, 111, 118 (Discussion: D-10, D-29, D-37, D-56, D-60, D-70)
Soil Mechanics and Foundations	43, 44, 48, 94, 102, 103, 106, 108, 109, 115 (Discussion: D-4, D-XXVIII, D-7, D-43, D-44, D-56)
Structural	42, 49, 51, 53, 54, 59, 61, 66, 89, 100, 103, 109, 113, 116, 117, 119, 121, 122, 123, 124, 125, 126 (Discussion: D-3, D-5, D-8, D-13, D-16, D-17, D-21, D-23, D-24, D-25-K, D-32, D-33, D-34, D-37, D-39, D-42, D-49, D-51, D-53, D-54, D-59, D-61, D-72)
Surveying and Mapping	50, 52, 55, 60, 63, 65, 68, 121 (Discussion: D-60, D-65)
Waterways	41, 44, 45, 50, 56, 57, 70, 71, 96, 107, 112, 113, 115, 120, 123 (Discussion: D-8, D-9, D-19, D-27, D-28, D-56, D-70, D-71)

A constant effort is made to supply technical material to Society members, over the entire range of possible interest. Insofar as your specialty may be covered inadequately in the foregoing list, this fact is a gage of the need for your help toward improvement. Those who are planning papers for submission to "Proceedings-Separates" will expedite Division and Committee action measurably by first studying the ASCE "Guide for Development of Proceedings-Separates" as to style, content, and format. For a copy of this Manual, address the Manager, Technical Publications, ASCE, 33 W. 39th Street, New York 18, N. Y.

The Society is not responsible for any statement made or opinion expressed in its publications

Published at Prince and Lemon Streets, Lancaster, Pa., by the American Society of Civil Engineers. Editorial and General Offices at 33 West Thirty-ninth Street, New York 18, N. Y. Reprints from this publication may be made on condition that the full title of paper, name of author, page reference, and date of publication by the Society are given.

Proceedings—Vol. 78 April, 1952 Separate No. 124

AMERICAN SOCIETY OF CIVIL ENGINEERS

Founded November 5, 1852

PAPERS

INFLUENCE LINES BY CORRECTIONS TO AN ASSUMED SHAPE

By James P. Michalos,[1] A. M. ASCE, and Edward N. Wilson,[2] J. M. ASCE

Synopsis

An analytical procedure is presented for obtaining influence lines for continuous structures by successive numerical corrections to an assumed shape. The structure may have any variation in cross section and may or may not be subject to sidesway. Any degree of precision is possible.

Introduction

In the design and analysis of continuous structures, influence lines are virtually essential. The usual method of constructing influence diagrams is by computing influence values for different positions of a unit load applied to the structure. This is rather tedious work, and the method is not particularly suitable to quick approximations which may be adequate for design purposes.

A procedure is presented in this paper for determining influence lines for any continuous girder or rigid frame by successively correcting an assumed shape. It can be used to obtain approximate influence lines for preliminary design or to obtain precise influence lines for final design and analysis. Certain factors related to those used in moment distribution[3] are employed, as are certain values generally available in tabular or graphical form. The procedure is based on two concepts:

1. The Müller-Breslau influence line theorem;[4] and
2. A concept by which the influence line for a continuous structure is obtained by adding corrections to an influence line obtained by neglecting continuity.

Note.—Written comments are invited for publication; the last discussion should be submitted by October 1, 1952.

[1] Associate Prof. of Civ. Eng., Iowa State College, Ames, Iowa.

[2] Structural Engr., Fruin-Colnon Contracting Co., St. Louis, Mo.

[3] "Analysis of Continuous Frames by Distributing Fixed-End Moments," by Hardy Cross, *Transactions*, ASCE, Vol. 96, 1932, p. 1.

[4] "Die Graphische Statik der Bankonstruktionen," by H. Müller-Breslau, 5th Ed., Vol. 2, Part 1, A. Kroner, Stuttgart, Germany, 1922.

The procedure involves an assumption of a statically possible solution. Such an assumption introduces errors in the geometry which are then adjusted, to any required precision, by successive corrections.

General Procedure

A powerful means for visualizing the shape of any influence line results from a principle stated by H. Müller-Breslau.[4] From this principle it follows that, if a unit deformation corresponding to the particular function—such as moment, shear, or reaction—under consideration is produced, the deflected structure draws its own influence line. By means of this concept influence lines may be quickly sketched. However, to obtain quantitative results it is necessary to evaluate individual ordinates to the line. This is usually done by computing the value of the particular function under consideration for different positions of a unit load.

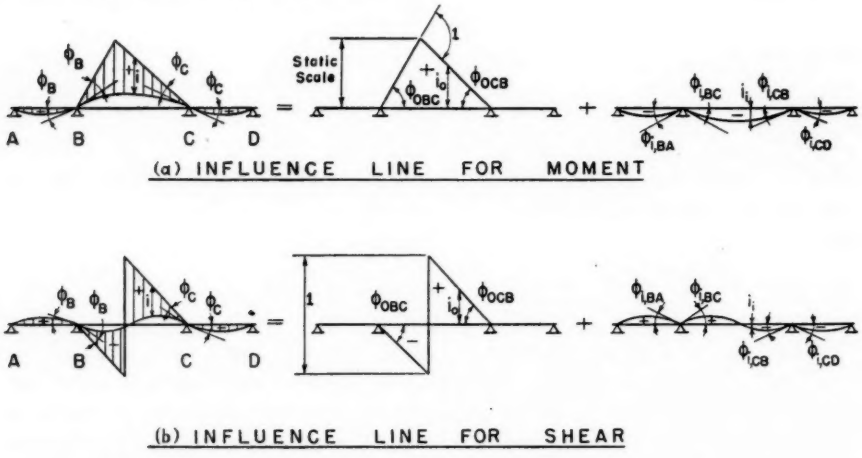


Fig. 1.—Influence Lines by Combining an Assumed Solution and a Correction

A quantitative scale can be obtained by constructing the influence diagram in two steps. First, the structure is considered cut so that it becomes statically determinate, and the influence line is drawn for the statically determinate span. Ordinates to this line can be determined exactly by computations of statics or from a consideration of the geometry of the bent line resulting from the application of a unit deformation. Secondly, correction deformations necessary to restore continuity are applied. If the deflected load line resulting from these correction deformations is superimposed on the statical influence line, an influence diagram for the continuous structure is obtained. Superposition to obtain influence lines, using computed influence ordinates, has been demonstrated elsewhere.[5]

Fig. 1 shows influence lines obtained in two steps. A solution which satisfies statics is assumed; but this assumption involves errors in the geometry, and a

[5] "Continuous Frames of Reinforced Concrete," by Hardy Cross and N. D. Morgan, John Wiley & Sons, Inc., New York, N. Y., 1932, pp. 249–252.

correction pattern is added such that the requirements of continuity are also satisfied. End slopes, ϕ, and ordinates, i, are obtained from

$$\phi = \phi_o + \phi_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1a)$$

and

$$i = i_o + i_i \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots (1b)$$

The subscripts o and i are associated with the assumed and the correction values, respectively. The angles will be considered positive when they are measured clockwise from the original position of the load line. The ordinates will be considered positive when they are on one side of the original position of the load line and negative when they are on the other side.

In Fig. 1 the unbalanced angle of the assumed static solution splits into the final angle and the correction angle. Thus, to restore continuity it is necessary to rotate the cut ends through a total relative angle equal to the angle of the assumed solution. In other words, the arithmetic sum of the correction angles at a joint is always known. The magnitude of the individual correction angles will depend on the relative flexibilities of the adjacent parts of the structure. By splitting the unbalanced angles, as best as one can, in proportion to the relative flexibilities, fairly accurate results are possible. To obtain precise results it is necessary to determine the correction angles and the corresponding displacements precisely. In what follows, first a method is presented for determining the correction angles to any desired accuracy, and then a method is presented for obtaining correction ordinates to a precision of the same order as that of the angles.

Determination of Correction Angles.—The method of moment distribution for determining moments at the joints of continuous structures is universally known. Since end slopes and moments are related, angle distribution has been proposed for use in analyzing continuous structures.[6] In addition, an angle distribution procedure has been used to construct an influence line for a beam of constant moment of inertia.[7]

In this section a perfectly general distribution method is presented for determining correction angles, ϕ_i. The structure is considered cut at all supports, and the correction angles are obtained by successively distributing unbalanced angles. Unbalanced angles are distributed in proportion to relative flexibilities, F, and each distributed angle induces an angle, called the carry-over angle, at the far end of the member. The carry-over angles are determined by multiplying the distributed angles by angle carry-over factors, C.

Suppose that a discontinuity, ϕ_o, is introduced by member AO at joint O of the frame in Fig. 2. To restore continuity at the joint, member AO and the remainder of the frame as a whole must be rotated relative to each other until the discontinuity is removed, that is, until all members framing into joint O have the same slope, ϕ. The stiffnesses, K, in Fig. 2 represent the moment necessary to rotate a member through a unit angle, the far end being hinged or fixed, as the case may be. For example, K_{OA} is the moment necessary to

[6] "Analysis of Continuous Frames by Balancing Angle Changes," by L. E. Grinter, *Transactions*, ASCE, Vol. 102, 1937, p. 1020.

[7] "Determining Influence Lines by Balancing Angle Changes," by L. E. Grinter, *Engineering News-Record*, Vol. 121, 1938, p. 443.

rotate member OA through a unit angle at joint O, end A being hinged, as shown. Flexibility is defined as the rotation per unit moment, or I/K. An unbalanced angle ϕ_o at joint O, Fig. 2, is distributed among the members of the joint as follows: ϕ_{iOA} to the member introducing the discontinuity, and ϕ to all other members. Since the sum of the moments at joint O must equal zero,

$$K_{OA}\,(\phi_o - \phi) = (K_{OB} + K_{OC} + K_{OD})\,\phi \quad \ldots\ldots\ldots\ldots (2)$$

from which

$$\phi = \frac{F_{OBCD}}{F_{OA} + F_{OBCD}}\,\phi_o \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (3)$$

Also in Fig. 2

$$\phi_{i\,OA} = -\,(\phi_o - \phi) = -\,\frac{F_{OA}}{F_{OA} + F_{OBCD}}\,\phi_o \quad \ldots\ldots\ldots\ldots (4)$$

In Eqs. 2, 3, and 4, $F_{OA} = \dfrac{1}{K_{OA}}$ expresses the flexibility of member OA at end O; and $F_{OBCD} = \dfrac{1}{K_{OB} + K_{OC} + K_{OD}}$ defines the group flexibility of members OB, OC, and OD.

In distributing angles, joints are considered free to rotate, and it is necessary to modify the usual tabulated values of stiffness (end fixed) by multiplying such values by $(1 - C_{AB}\,C_{BA})$, in which C_{AB} and C_{BA} are moment carry-over factors from end A to end B and from end B to end A of a member, respectively. Values of stiffness, far end fixed, and of moment carry-over factors are generally available.[8]

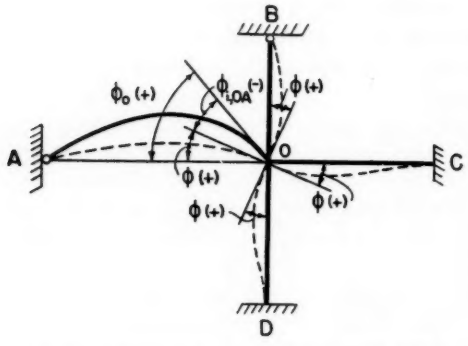


FIG. 2.—DISTRIBUTION OF AN UNBALANCED ANGLE

It can be shown that the angle carry-over factor, $\mathfrak{C}_{BA}$, from end B to end A of any member is always numerically equal, but opposite in sign, to the moment carry-over factor, C_{AB}, from end A to end B. That is,

$$\mathfrak{C}_{BA} = -\,C_{AB} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots (5)$$

Table 1 illustrates the method of determining correction angles in the general case of a continuous beam with a variable moment of inertia. Table 1(*a*) contains the constants used in distributing angles, and the moment distribution constants from which they were computed. Attention is called to the fact that a modified stiffness (stiffness, end hinged) was not used for member CD, in Col. 4. The distribution procedure is simplified if end D is considered fixed, as it actually is. If a modified stiffness were used for CD, it would be necessary to restore the angle at D to zero after each carry-over.

[8] "Handbook of Frame Constants," Portland Cement Assn., Chicago, Ill., 1947.

Inasmuch as only two members meet at each joint, the correction angles to be distributed are directly proportional to the flexibilities of the individual members.

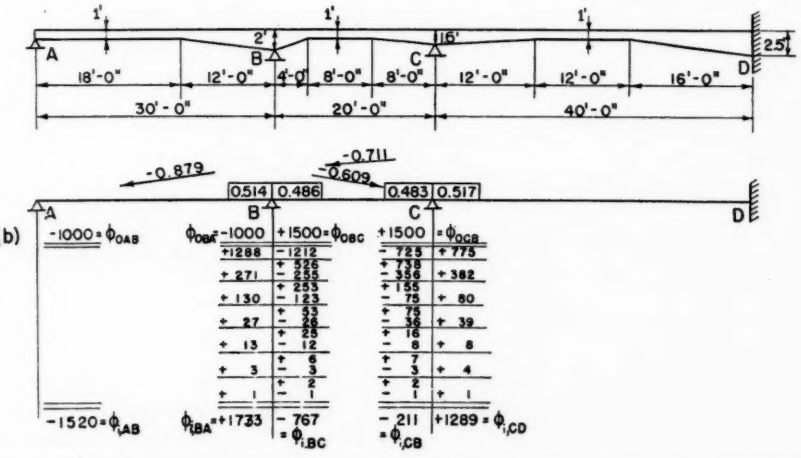
The actual distribution of unbalanced angles is shown in Table 1(*b*). Angle distribution factors are indicated in boxes above the joints, and angle carry-over factors are written above arrows indicating the direction of the carry-over.

TABLE 1.—CORRECTION ANGLES, CONTINUOUS BEAM WITH VARIABLE MOMENT OF INERTIA

(*a*) DISTRIBUTION CONSTANTS

Member	MOMENT DISTRIBUTION CONSTANTS			CONSTANTS FOR DISTRIBUTING ANGLES		
	Moment carry-over	Relative stiffness		Angle carry-over	Relative flexibility	Distribution factor
		Fixed end	Hinged end			
(1)	(2)	(3)	(4)	(5)	(6)	(7)
AB	0.879	0.162	...	...	...	...
BA	0.418	0.341	0.216	−0.879	4.63	0.514
BC	0.711	0.400	0.227	−0.609	4.40	0.486
CB	0.609	0.467	0.265	−0.711	3.77	0.483
CD	0.907	0.248	...	...	4.03	0.517
DC	0.562	0.400	...	...	...	...

(*b*) DISTRIBUTION OF UNBALANCED ANGLES



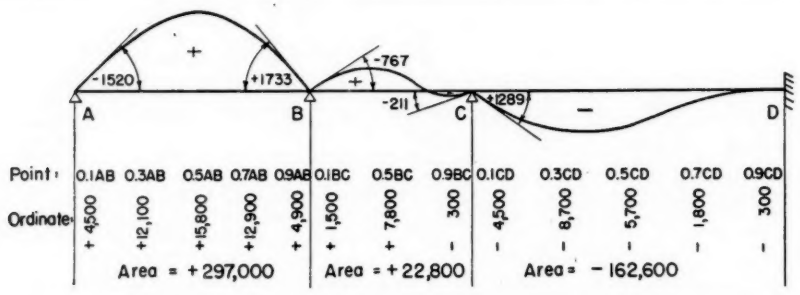
For purposes of illustration, unbalanced angles of − 1,000 at each end of member AB and + 1,500 at each end of span BC have been assumed. These values have been underscored twice to separate them from the successive corrections. Signs are determined automatically. As previously explained, angles are considered positive when they are measured clockwise from the original position. The distribution to the member introducing the unbalanced

angle is opposite in sign to that angle; whereas the distribution to the other member (or members) is of the same sign as the unbalanced angle. Because end A is hinged, no carry-over would be made from end A to end B. Consequently, it is not necessary to make intermediate carry-overs from B to A.

Determination of Correction Ordinates.—Consider an influence line for a fixed-end moment. The ordinates to such an influence line, for members of various shapes, can be found in many available tables and charts,[8] but from the Müller-Breslau principle, these ordinates are displacements of the load line that result from the application of a unit rotation at the end of the member,

TABLE 2.—ORDINATES OF THE DEFLECTED LOAD LINE WHEN END ANGLES ARE KNOWN

(*a*) DEFLECTION ORDINATES FROM END ANGLES



(*b*) DETAIL COMPUTATIONS, SPAN AB

Description	0.1 $\overline{AB}$	0.3 $\overline{AB}$	0.5 $\overline{AB}$	0.7 $\overline{AB}$	0.9 $\overline{AB}$	Area
(1)	(2)	(3)	(4)	(5)	(6)	(7)
Influence ordinate for M_{AB}[a]	2.32	3.63	2.44	0.85	0.07	54.6
Influence ordinate for M_{BA}[a]	0.56	3.81	6.98	6.70	2.77	123.7
1,520 × influence ordinates for M_{AB}	3,520	5,500	3,700	1,300	100	83,000
1,733 × influence ordinates for M_{BA}	980	6,600	12,100	11,600	4,800	214,000
Deflection ordinate	4,500	12,100	15,800	12,900	4,900	297,000

[a] Their values are available in various publications.

the far end being fixed. Hence, displacements resulting from any end angle may be computed by ratio from influence values for fixed-end moments. If there are rotations at both ends of a member, the displacements resulting from each end angle can be computed separately and combined to obtain the actual displacement. Similarly, areas under the deflected line may be computed by multiplying the tabulated fixed-end moment for a unit uniform load by the actual end angles. For the special case of members of constant moment of inertia, ordinates to the influence line for fixed-end moment at an end A may be determined readily by means of the expression:

$$i = a\,b^2\,L \qquad (6)$$

in which: L = length of member; $a\,L$ = distance from end A to the section considered; and $b\,L$ = distance from section considered to opposite end of member. The area under such an influence line is $L^2/12$.

Table 2 illustrates the computation of ordinates to the deflected load line when the end angles are known. The beam is that of Table 1. Detail computations are shown in Table 2(*b*) for span AB. At each point the ordinate to

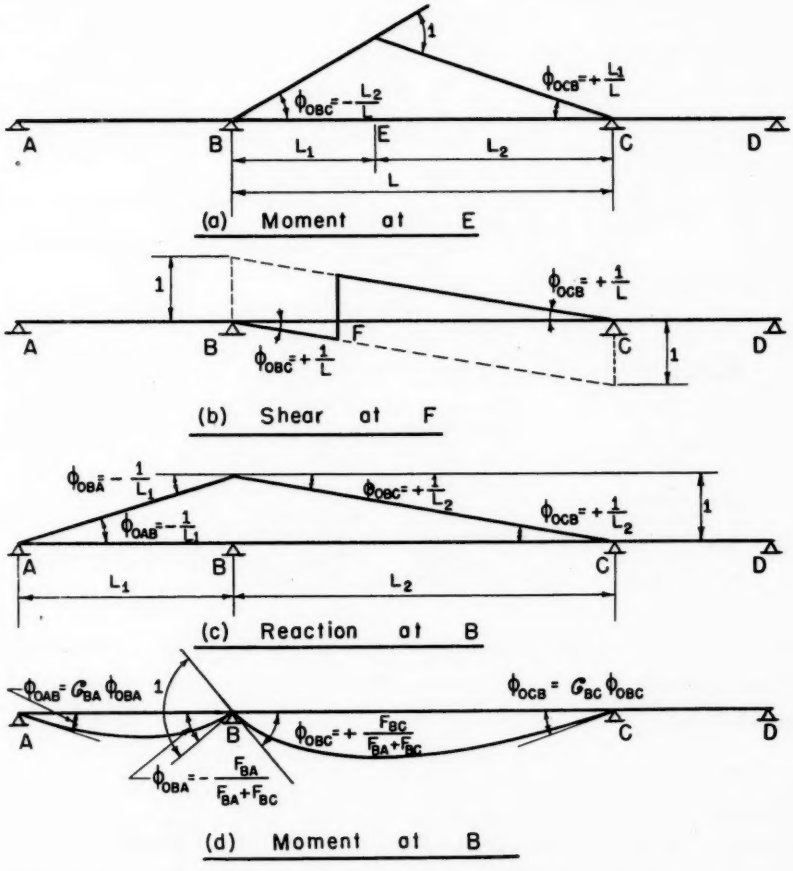


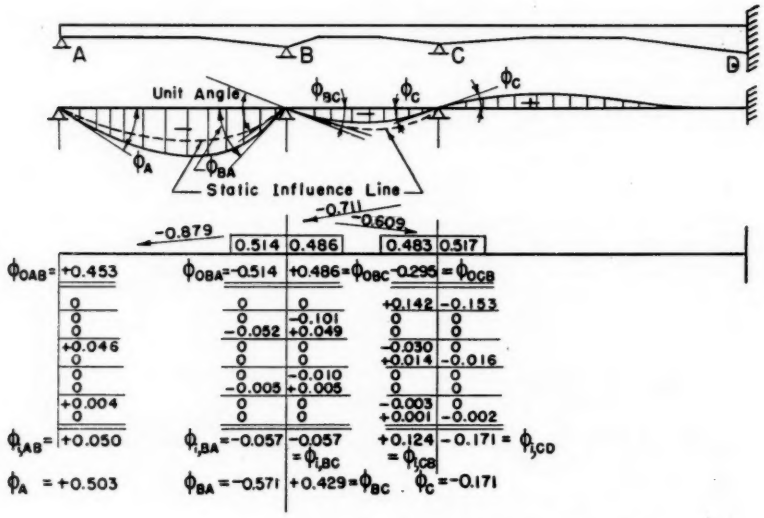
FIG. 3.—STATIC INFLUENCE LINES

the deflected load line is obtained by ratio from the values for fixed-end moments at ends A and B. The latter values are readily available in various publications. In span BC the rotation at end C produces displacements opposite to those produced by the rotation at end B. As a result, ordinates and areas are computed as the numerical difference of those found for rotations at ends B and C separately.

Influence Lines For Continuous Beams

In this section the step-by-step procedure, as applied to continuous beams, is outlined, and several numerical examples are presented.

The first step is to consider the structure cut at the supports. Next draw the influence line for the statically determinate structure, and obtain the unbalanced angles, ϕ_o, at the joints. These angles can be determined readily by substitution in the expressions shown on the statical influence lines in Fig. 3. In each case the expressions result from the application, to the statically determinate structure, of a unit deformation corresponding to the function under consideration. The correction angles, ϕ_i, which are to be added to the



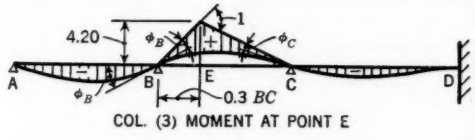
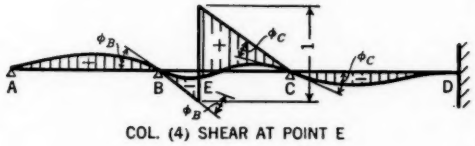
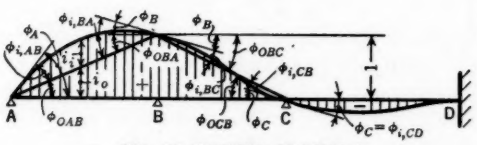
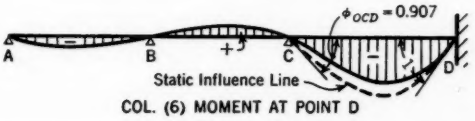
Point:	0.3AB	0.5AB	0.7AB	0.3BC	0.5BC	0.7BC	0.3CD	0.5CD	0.7CD
Influence Ordinate:	-3.97	-5.22	-4.27	-1.89	-1.85	-1.16	+1.14	+0.76	+0.23
Influence Area:		-98.1			-23.9			+21.6	

Fig. 4.—Influence Line for Moment at Point B

unbalanced angles are then computed, to any desired accuracy, as in Table 1. With the correction angles known, the correction influence ordinates, i_i, are computed and added to the static influence ordinates, i_o, to obtain the influence ordinates for the continuous beam.

Table 3 and Fig. 4 show influence lines for moment, shear, and reaction in the same continuous beam as that in Table 1. In each case the correction angles and ordinates were determined as in Tables 1(*b*) and 2. Areas under the influence line in each span were determined as explained in the previous section and illustrated in Table 2. In the influence line for shear at point E (values in Table 3, Col. 4) it is of particular interest to note that the original unbalanced angles at points B and C (ϕ_{OBC} and ϕ_{OCB}) will be the same for all

TABLE 3.—Influence Lines for Continuous Beams with Variable Moment of Inertia

Influence line	Joint	Point E		Reaction point B	Moment point D
		Moment	Shear		
(1)	(2)	(3)	(4)	(5)	(6)
 COL. (3) MOMENT AT POINT E	(a) Unbalanced Angles ϕ_o (That is, ϕ_{OAB}, ϕ_{OBA}, etc.)				
	AB	...	...	−0.0333	...
	BA	...	...	−0.0333	...
	BC	−0.700	+0.0500	+0.0500	...
	CB	+0.300	+0.0500	+0.0500	...
	CD	...	...	...	+0.907
	DC	...	...	...	−1.000
 COL. (4) SHEAR AT POINT E	(b) Correction Angles ϕ_i (That is, $\phi_i AB$, $\phi_i BA$, etc.)				
	AB	+0.301	−0.0338	−0.0507	+0.157
	BA	−0.342	+0.0385	+0.0577	−0.179
	BC	+0.358	−0.0115	−0.0256	−0.179
	CB	−0.246	−0.0127	−0.0070	+0.386
	CD	+0.054	+0.0373	+0.0430	−0.521
	DC	0	0	0	0
 COL. (5) REACTION AT POINT B	(c) Final Angles (That is, ϕ_D, ϕ_C, ϕ_B, and ϕ_A, respectively)				
	D	0	0	0	−1.000
	C	+0.054	+0.0373	+0.0430	+0.386
	B	−0.342	+0.0385	+0.0244	−0.179
	A	+0.301	−0.0338	−0.0840	+0.157
 COL. (6) MOMENT AT POINT D					

(d) Influence Ordinates

Column[a]	Span A to B				Span B to C				Span C to D			
	0.3 $\overline{AB}$	0.5 $\overline{AB}$	0.7 $\overline{AB}$	Area[b]	0.3 $\overline{BC}$	0.5 $\overline{BC}$	0.7 $\overline{BC}$	Area[b]	0.3 $\overline{CD}$	0.5 $\overline{CD}$	0.7 $\overline{CD}$	Area[b]
3	−2.40	−3.12	−2.55	−58.6	+2.50	+1.14	+0.45	+18.1	−0.36	− 0.25	−0.07	− 6.8
4	+0.27	+0.35	+0.23	+ 6.58	−0.08[c]	+0.50	+0.08[c]	+ 4.00	−0.25	− 0.17	−0.05	− 4.70
5	+0.70	+1.02	+1.13	+24.90	+0.79	+0.56	+0.30	+10.76	−0.29	− 0.19	−0.06	− 5.42
6	−1.25	−1.63	−1.33	−30.80	+1.16	+1.77	+1.64	+22.2	−6.71	−11.07	−9.86	−262.8

[a] Column numbers on Tables (a), (b), and (c). [b] Influence area. [c] These are the shear influence ordinates for 0.1 $\overline{BC}$ and 0.9 $\overline{BC}$.

shear influence lines in span BC. As a result, the correction angles and ordinates are identical for all shear influence lines. In the influence line for moment at D (values in Table 3, Col. 6) the required unit deformation at point D is automatically retained during the distribution of angles. These influence ordinates are obtained directly from the final angles. In Fig. 4, the unit

angular deformation introduced at point B, with ends A and C free to rotate, is split between spans AB and BC in the manner indicated in Fig. 3(*d*). The values thus obtained—0.514 to BA and + 0.486 to BC—are the only ones that satisfy statics. The unbalanced angles at points A and C also have been

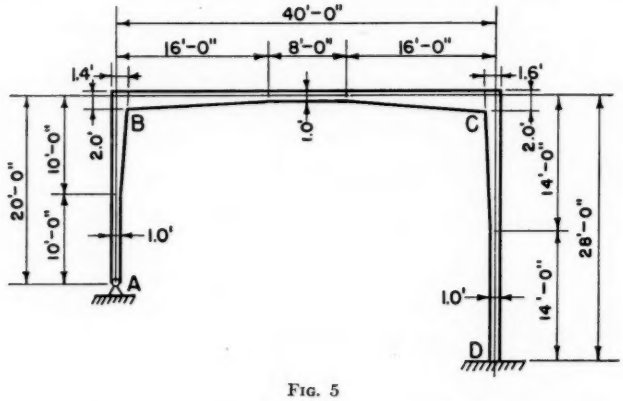


FIG. 5

determined from the relationships shown in Fig. 3(*d*). Since the angles at joint B are already balanced, zero angle is distributed at this joint in the first cycle. Carry-overs and distributions then follow as before.

TABLE 4.—FRAME CONSTANTS USED IN DETERMINING PRECISE INFLUENCE LINES (REFER FIG. 5)

Member	Moment Distribution Constants[a]			Angle Distribution Constants[b]			Influence Values for Fixed-End Moments[a]			
	Carry-over factor	Relative stiffness		Carry-over factor	Flexibility	Distribution factor	Points[c]			Influence area
		Fixed end	Hinged end				0.3 *L*	0.5 *L*	0.7 *L*	
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)
AB	0.697	0.222	...	...	...	...	2.69	2.06	0.90	28.72
BA	0.434	0.356	0.248	−0.697	4.03	0.41	1.86	3.55	3.62	43.16
BC	0.720	0.357	0.172	−0.720	5.95	0.59	7.96	6.80	2.60	167.36
CB	0.720	0.357	0.172	−0.720	5.95	0.65	2.60	6.80	7.96	167.36
CD	0.413	0.314	....	−0.788	3.18	0.35	5.46	5.48	3.02	52.7
DC	0.788	0.165	0.111	−0.413	...	...	1.07	2.63	3.62	93.3

[a] These values are generally available. [b] These are derived values. [c] *L* denotes any corresponding member shown in Col. (1).

INFLUENCE LINES FOR RIGID FRAMES

As with continuous girders, influence lines for rigid frames are obtained by adding a correction pattern to a static influence line. In general, however, this correction pattern must be adjusted for sidesway induced in the frame.

Fig. 5 shows an unsymmetrical frame, hinged at one end and fixed at the other. The constants necessary for determining the correction angles are

shown in Table 4. Influence values for fixed-end moment, used in computing the ordinates and areas of the correction patterns, are included also. The moment distribution constants and the influence values for fixed-end moments

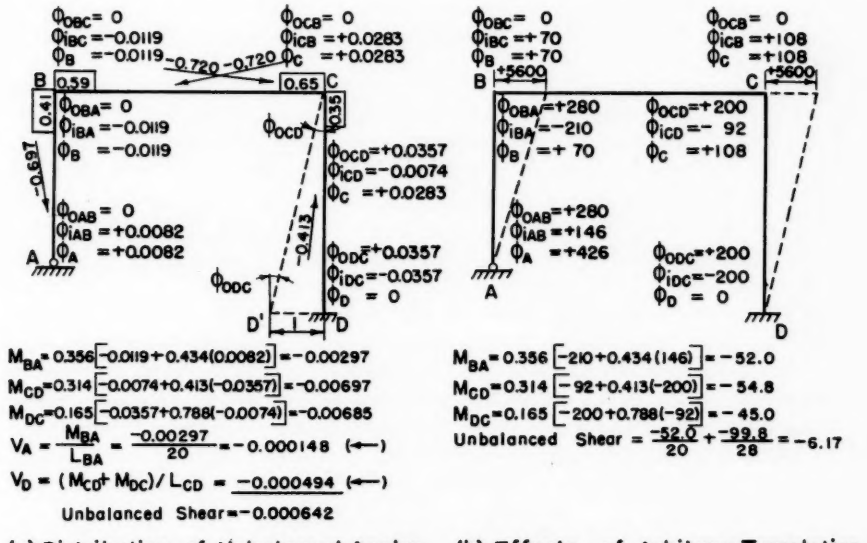


FIG. 6.—INFLUENCE LINE FOR HORIZONTAL REACTION

can be found in various publications.[8] The calculations for determining the influence line for horizontal reaction at point D are shown in Fig. 6. In Fig. 6(*a*) the static influence line is shown dashed, and the numerical values of the original unbalanced angles, the correction angles, and the final angles are given.

It can be shown that in any member, AB subjected to rotations ϕ_A and ϕ_B at its ends A and B, respectively,

$$M_{AB} = K_{AB} (\phi_A + C_{AB} \phi_B) \quad (7a)$$

and

$$M_{BA} = K_{BA} (\phi_B + C_{BA} \phi_A) \quad (7b)$$

in which K_{AB} and K_{BA} are stiffnesses, A to B and B to A, respectively, with the far end fixed. The calculation of such moments is shown in Fig. 6(*a*). The end shears are then obtained, as shown, and it is found that they do not balance. To satisfy statics a force must be applied along member BC so as to prevent joints B and C from moving to the left. If this force can be supplied by external restraint to movement, the computed angles are correct; but, if no such restraint is available, the frame will sway and the angles must be adjusted accordingly. In the latter case it is necessary to superimpose on the rotations in Fig. 6(*a*) additional rotations resulting from the lateral movement of joints B and C.

In Fig. 6(*b*) an arbitrary translation of joints B and C, with numerical value picked for convenience only, is introduced. It has been introduced intentionally in the wrong direction and is called a positive translation because it is accompanied by positive rotations. The unbalanced angles resulting from the arbitrary translation are computed, and, as before, correction angles and final angles are determined. The shear, − 6.17, is then obtained, as in Fig. 6(*a*).

In Fig. 6(*c*) (which shows the influence line), the actual translation and the accompanying joint rotations are shown. Since an assumed translation of 5,600 results in a shear of − 6.17, the correction translation compatible with the unbalanced shear in Fig. 6(*a*) is

$$-\frac{0.000642}{6.17}(5{,}600) = -0.583.$$

The minus sign corrects for the fact that the arbitrary translation was taken in the wrong direction. The joint rotations accompanying sidesway are obtained in a similar manner. For example, the rotation at point A is $-\frac{0.000642}{6.17}(426) = -0.0443$. The final slopes of the influence line are then determined by adding to the angles in Fig. 6(*a*) those due to sidesway. For example, $\phi_A = +0.0082 - 0.0443 = -0.0361$.

In the influence line shown in Fig. 6(*c*) all ordinates that fall on the outside periphery of the structure have the same sign, and those on the inside have the opposite sign. These signs are consistent when the loads considered are applied inwardly or when the loads are applied outwardly. Thus the horizontal reaction at point D is to the left when loads are applied either to the right on leg AB or downward on girder BC. On the other hand, the horizontal reaction is in the opposite direction when a load is applied inwardly (that is, to the left) on leg CD. The influence ordinates along the legs were determined by adding the ordinates due to translation only to those due to rotation in the translated position. The values of the end angles required to compute that increment of

an influence ordinate due to rotation in the translated position can be determined readily by means of the final end slopes and translations. For example, at point C the angle is $0.0171 - \frac{1.000 - 0.583}{28} = 0.0022$.

Influence lines have been obtained for moment in the girder and for moment at end D of the frame in Fig. 5 but are not included in this paper. The procedure is similar to that of Fig. 6. The effects of an arbitrary translation have already been determined in Fig. 6(*b*). These results are used in correcting all other influence lines for the effects of sidesway. In determining an influence line for moment at point D, after the original unbalanced angles ($\phi_{ODC} = -1.000$; and $\phi_{OCD} = +0.413$) are distributed, the moment, M_{ODC}, necessary to impose the original unit deformation (rotation) at point D of the cut structure is included in computing the shear at point D. That is, the shear at point D is computed as $(M_{ODC} + M_{DC} + M_{CD}/28)$.

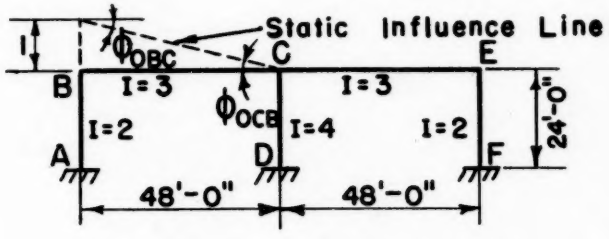


FIG. 7.—PROPERTIES OF A TWO-SPAN RIGID FRAME

The final influence line presented is for the vertical reaction in a three-legged bent of prismatic members. Properties of such a frame are shown in Fig. 7. Relative stiffness (hinged end for the girders; but for the actual fixed-end condition of the columns) were computed as follows: $\frac{3}{4} \times \frac{3}{48} = 0.0468$ for the girders; $\frac{2}{24} = 0.0833$ for the outside columns; and $\frac{4}{24} = 0.1667$ for the interior column. Since there are more than two members framing into joint C, more than one set of distribution factors is required. These factors were computed by means of Eqs. 3 and 4 and are shown boxed in Fig. 8(*a*). The first number in the box represents the percentage of the unbalanced angle to be distributed to a particular member when the unbalance is introduced by that member. The number in parenthesis represents the percentage to be distributed to all other members framing into joint C. Because the flexibilities of the columns were determined for the actual condition of fixed ends, no angles are carried over to points A, D, or F. The unbalanced shear is determined as before.

In Fig. 8(*b*) the shear due to an arbitrary translation of + 24,000 is determined. The actual translation of the column tops is $\frac{0.000116}{26.9} \times 24{,}000$

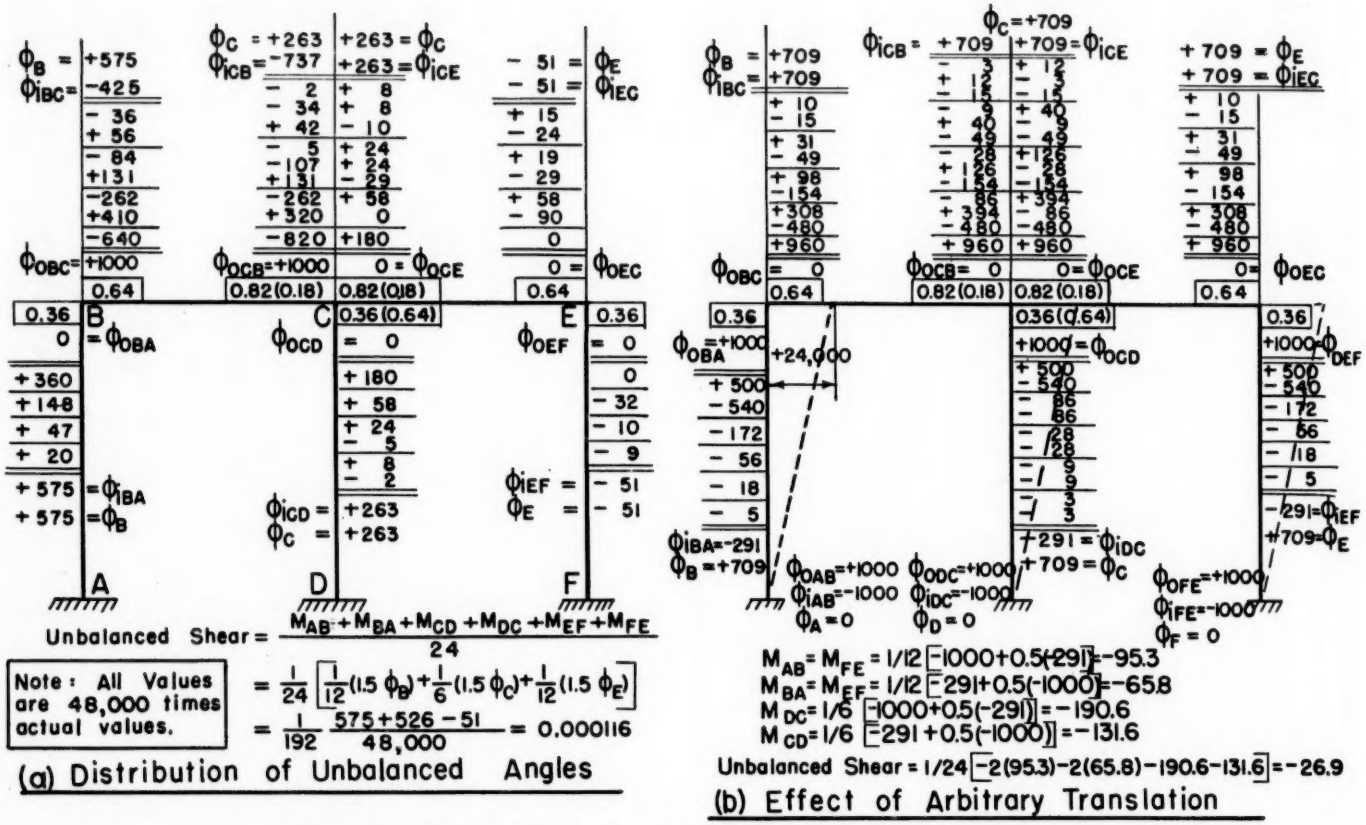

(a) Distribution of Unbalanced Angles
Note: All Values are 48,000 times actual values.
Unbalanced Shear = (MAB + MBA + MCD + MDC + MEF + MFE) / 24
= 1/24 [1/12 (1.5 φB) + 1/6 (1.5 φC) + 1/12 (1.5 φE)]
= 1/192 (575 + 526 − 51) / 48,000 = 0.000116
(b) Effect of Arbitrary Translation
MAB = MFE = 1/12 [−1000 + 0.5(−291)] = −95.3
MBA = MEF = 1/12 [−291 + 0.5(−1000)] = −65.8
MDC = 1/6 [−1000 + 0.5(−291)] = −190.6
MCD = 1/6 [−291 + 0.5(−1000)] = −131.6
Unbalanced Shear = 1/24 [−2(95.3) − 2(65.8) − 190.6 − 131.6] = −26.9

FIG. 8.—DISTRIBUTION PROCEDURE, INFLUENCE LINE FOR VERTICAL REACTION AT POINT A

$= 0.1035$; the angles caused by the translation are (at points B, C, and E) $+ 709 \times \frac{0.1035}{24{,}000} = + 0.0031$ and, before translation, the same angles were: At point B, $\frac{+ 575}{48{,}000} = + 0.0120$; at point C, $\frac{+ 263}{48{,}000} = + 0.0055$; and, at point E, $\frac{- 51}{48{,}000} = - 0.0011$. The final angles are: $\phi_B = + 0.0120 + 0.0031 = + 0.0151$; $\phi_C = + 0.0055 + 0.0031 = + 0.0086$; and $\phi_E = - 0.0011 + 0.0031 = + 0.0020$. The influence line is shown in Fig. 9. The sign

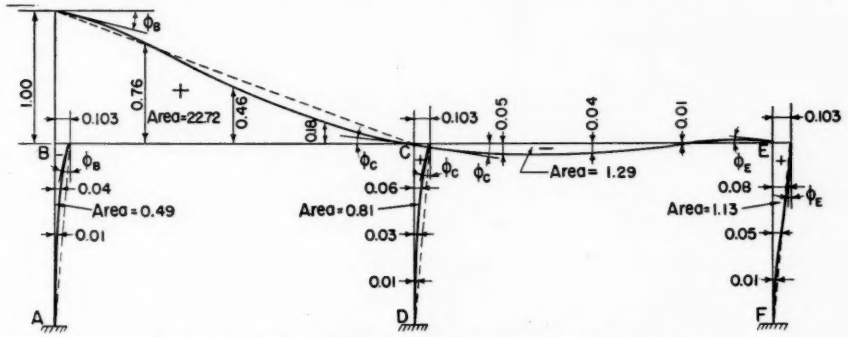


FIG. 9.—INFLUENCE LINE FOR VERTICAL REACTION AT POINT A

convention used for ordinates is the same as that explained in connection with Fig. 6(*c*). The leg CD has been considered an outside member of circuit ABCD. As before, the correction influence ordinates, i_i, have been obtained by ratio from the ordinates to influence lines for fixed-end moment.

OTHER APPLICATIONS

The procedure described in this paper can be used to obtain preliminary influence lines for the design of continuous trusses. The trusses may be treated as continuous beams, with or without variation in moment of inertia, and influence lines may be determined for moment at the supports. By means of these influence lines, influence diagrams may then be obtained for stress in the various members.

If desired, the procedure can also be used to determine displacements in continuous beams and frames. The ends of all members are considered cut, and the deflections and end rotations of all loaded members are determined. Then correction angles and ordinates are computed and combined with the angles and ordinates of the assumed static solution to obtain the true deflected structure.

CONCLUDING REMARKS

A procedure has been presented for obtaining influence lines, to any desired precision, directly from a consideration of geometry. The structure for which influence lines are desired may have members of constant or variable moment of inertia and may or may not be subject to sidesway. In addition to enabling one to determine precise influence values, the procedure readily lends itself to the development of judgment and a sense of scale.

CURRENT PAPERS AND DISCUSSIONS

Proceedings-Separate Number	Date Available	Title and Author	Discussion closes
90	Nov., 1951	"Consumptive Use of Water by Forest and Range Vegetation," by L. R. Rich	Apr. 1
91	Nov., 1951	"Consumptive Use of Water," by Harry F. Blaney	Apr. 1
92	Nov., 1951	"Experimental Investigation of Fire Monitors and Nozzles," by Hunter Rouse, J. W. Howe, and D. E. Metzler	Apr. 1
93	Nov., 1951	"Aircraft Design as Related to Airport Standards," by Milton W. Arnold	Apr. 1
94	Nov., 1951	"Friendship International Airport," by Benjamin Everett Beavin	Apr. 1
95	Nov., 1951	"Directional Requirements for Airport Runways," by Ralph H. Burke and Harry Otis Wright, Jr.	Apr. 1
96	Nov., 1951	"Surface Curves for Steady Nonuniform Flow," by Robert B. Jansen	Apr. 1
97	Dec., 1951	"Consumptive Use in the Rio Grande Basin," by Robert L. Lowry	May 1
98	Dec., 1951	"Consumptive Use of Water on Irrigated Land," by Wayne D. Criddle	May 1
99	Dec., 1951	"Consumptive Use in Municipal and Industrial Areas," by George B. Gleason	May 1
100	Dec., 1951	"Forced Vibrations of Continuous Beams," by Edward Saibel and Elio D'Appolonia	May 1
101	Dec., 1951	"Application of Highway Capacity Research," by J. P. Buckley	May 1
102	Dec., 1951	"Utilization of Ground Water in California," by T. Russel Simpson	May 1
103	Dec., 1951	"Pile Foundations for Large Towers on Permafrost," by L. A. Nees	May 1
104	Dec., 1951	"Redesign of Major Airport Terminals," by Herbert H. Howell	May 1
105	Dec., 1951	"Principles of Highway Capacity Research," by O. K. Normann	May 1
106	Jan., 1952	"Analysis of Ground-Water Lowering Adjacent to Open Water," by Stuart B. Avery, Jr.	June 1
107	Jan., 1952	"Characteristics of the Solitary Wave," by James W. Daily and Samuel C. Stephan, Jr.	June 1
108	Jan., 1952	"Control of Embankment Material by Laboratory Testing," by F. C. Walker and W. G. Holtz	June 1
109	Jan., 1952	"Final Foundation Treatment at Hoover Dam," by A. Warren Simonds	June 1
110	Jan., 1952	"Review of Flood Frequency Methods," Final Report of the Subcommittee of the ASCE Joint Division Committee on Floods	June 1
111	Jan., 1952	"Research in Water Spreading," by Dean C. Muckel	June 1
112	Feb., 1952	"Diversions from Alluvial Streams," by C. P. Linder	July 1
113	Feb., 1952	"Wave Forces on Breakwaters," by Robert Y. Hudson	July 1
114	Feb., 1952	"Utilization of Underground Storage Reservoirs," by Harvey O. Banks	July 1
115	Mar., 1952	"Lake Michigan Erosion Studies," by Colonel John R. Hardin and William H. Booth, Jr.	Aug. 1
116	Mar., 1952	"Graphical Solution of Hydraulic Problems," by Kenneth E. Sorensen	Aug. 1
117	Mar., 1952	"The Development of Stresses in Shasta Dam," by J. M. Raphael	Aug. 1
118	Mar., 1952	"Flocculation Phenomena in Turbid Water Clarification," by W. F. Langelier, Harvey F. Ludwig, and Russell G. Ludwig	Aug. 1
119	Apr., 1952	"Thin-Walled Members in Combined Torsion and Flexure," by Warner Lansing	Sept. 1
120	Apr., 1952	"Surface Water Wave Theories," by Martin A. Mason	Sept. 1
121	Apr., 1952	"Rate of Change of Grade per Station," by Clarence J. Brownell	Sept. 1
122	Apr., 1952	"Engineering Aspects of Diffraction and Refraction," by J. W. Johnson	Sept. 1
123	May, 1952	"Long-Period Waves or Surges in Harbors," by John H. Carr	Oct. 1
124	May, 1952	"Influence Lines by Corrections to an Assumed Shape," by James P. Michalos and Edward N. Wilson	Oct. 1
125	May, 1952	"Torsion of Plate Girders," by F. K. Chang and Bruce G. Johnston	Oct. 1
126	May, 1952	"Variation of Wind Velocity and Gusts with Height," by R. H. Sherlock	Oct. 1

AMERICAN SOCIETY OF CIVIL ENGINEERS

OFFICERS FOR 1952

PRESIDENT

CARLTON S. PROCTOR

VICE-PRESIDENTS

Term expires October, 1952:

WILLIAM R. GLIDDEN
DANIEL V. TERRELL

Term expires October, 1953:

GEORGE W. BURPEE
A M RAWN

DIRECTORS

Term expires October, 1952:

MILTON T. WILSON
MORRIS GOODKIND

Term expires January, 1953:

OTTO HOLDEN
FRANK L. WEAVER
GORDON H. BUTLER
G. BROOKS EARNEST
GEORGE W. LAMB
EDWARD C. DOHM

Term expires October, 1953:

KIRBY SMITH
FRANCIS S. FRIEL
WALLACE L. CHADWICK
NORMAN R. MOORE
BURTON G. DWYRE
LOUIS R. HOWSON

Term expires October, 1954:

WALTER D. BINGER
FRANK A. MARSTON
GEORGE W. McALPIN
JAMES A. HIGGS
I. C. STEELE

PAST-PRESIDENTS

Members of the Board

ERNEST E. HOWARD
GAIL A. HATHAWAY

TREASURER
CHARLES E. TROUT

EXECUTIVE SECRETARY
WILLIAM N. CAREY

ASSISTANT TREASURER
GEORGE W. BURPEE

ASSISTANT SECRETARY
E. L. CHANDLER

PROCEEDINGS OF THE SOCIETY

SYDNEY WILMOT
Manager of Technical Publications

HAROLD T. LARSEN
Editor of Technical Publications

COMMITTEE ON PUBLICATIONS

LOUIS R. HOWSON

FRANCIS S. FRIEL
I. C. STEELE
OTTO HOLDEN
FRANK A. MARSTON
NORMAN R. MOORE